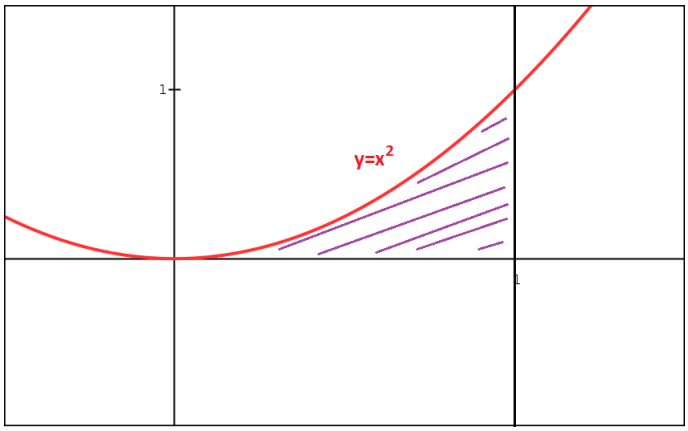
QUIZ 10 SOLUTIONS: LESSONS 13-14
SEPTEMBER 29, 2017

Write legibly, clearly indicate the question you are answering, and put a box or circle around your final answer. If you do not clearly indicate the question numbers, I will take off points. Write as much work as you need to demonstrate to me that you understand the concepts involved. If you have any questions, raise your hand and I will come over to you.

Let S be the region bounded by the curves

$$y = x^2, \quad y = 0, \quad x = 1.$$

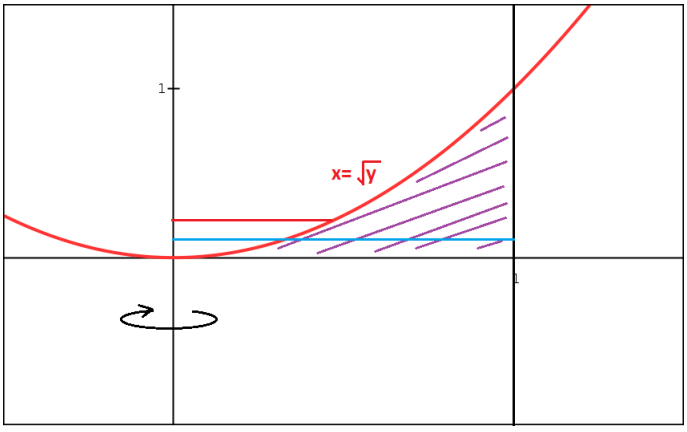
This region looks like



1. Find the volume of the solid obtained by revolving the region S about the y-axis.

   Solution:

   We have this picture



Because we are revolving around a vertical line, we must solve for x. We have

$$y = x^2 \Rightarrow x = \sqrt{y}.$$

Since there is a gap between where we are revolving and S, we use the washer method. Thus

$$\text{Outer Radius} : x = 1$$
$$\text{Inner Radius} : x = \sqrt{y}$$
$$\text{Bounds} : 0 \leq y \leq 1$$

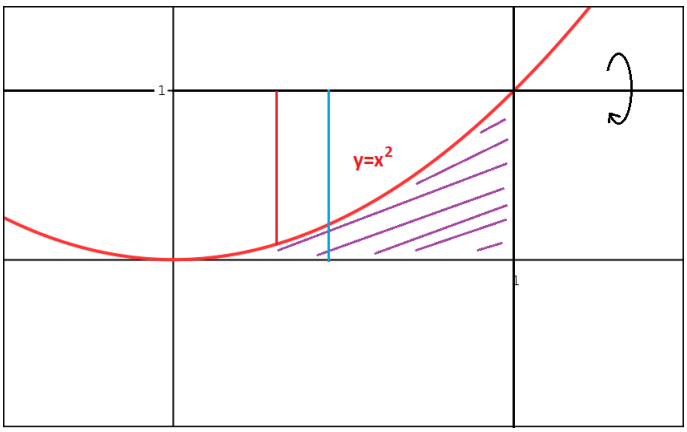
Putting this all together, our volume is given by

$$\begin{aligned}
\text{Volume} &= \int_0^1 \pi \left[(1)^2 - (\sqrt{y})^2\right] \, dy \\
&= \int_0^1 \pi \left[1 - y\right] \, dy \\
&= \pi \left[y - \frac{1}{2}y^2\right]_0^1 \\
&= \pi \left(1 - \frac{1}{2}(1)^2\right) \\
&= \boxed{\frac{\pi}{2}}
\end{aligned}$$

2. Find the volume of the solid obtained by revolving the region S about the line $y = 1$.

Solution:

We have this picture



This is a washer method problem because there is a gap between the S and where we are revolving. The outer radius is the **difference** between $y = 1$ and the x-axis and the inner radius is the **difference** between $y = 1$ and the

curve $y = x^2$. This means

$$\text{Outer Radius} : y = 1 - 0$$

$$\text{Inner Raidus} : y = 1 - x^2$$

$$\text{Bounds} : 0 \leq x \leq 1$$

Thus

$$\begin{aligned}
\text{Volume} &= \int_0^1 \pi \left[(1-0)^2 - \left(1 - x^2\right)^2 \right] \, dx \\
&= \int_0^1 \pi \left[1 - (1 - x^2)^2 \right] \, dx \\
&= \pi \int_0^1 \left[1 - (1 - 2x^2 + x^4) \right] \, dx \\
&= \pi \int_0^1 \left[2x^2 - x^4 \right] \, dx \\
&= \pi \left(\frac{2}{3}x^3 - \frac{1}{5}x^5 \right) \Big|_0^1 \\
&= \pi \left(\frac{2}{3}(1)^3 - \frac{1}{5}(1)^5 \right) \\
&= \pi \left(\frac{10}{15} - \frac{3}{15} \right) \\
&= \boxed{\frac{7\pi}{15}}
\end{aligned}$$